Regulation by Iterative Learning in Continuum Soft Robots

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***Abstract*—The dynamic uncertainties and disturbances characterizing continuum soft robots call for the derivation of simple and possibly information-free controllers. We propose an iterative learning control law for shape regulation of continuum soft robots consisting of a PD action and a feedforward term, updated to learn the potential forces at the target configuration. We prove that the regulator achieves global asymptotic stabilization of the closed-loop system to the desired set-point. Simulation results validate the proposed control law.**

***Index Terms*—Soft Robotics, Iterative Learning Control, Motion Control.**

I. Introduction

Continuum soft robots are robotic systems with continuously deformable bodies [1], which have potential applications ranging from maintenance of hostile or even inaccessible environments [2], [3] to human rehabilitation [4], [5].

Meeting these scenarios requires providing soft robots with the ability to perform, at the very least, elementary tasks, such as shape regulation. Unfortunately, the infinite dimensionality of these systems makes it non-trivial to achieve even such a primary skill. To overcome this issue, researchers proposed finite-dimensional models specifically designed for control purposes [6], [7]. These formulations allowed applying, with little effort, many control laws designed for other mechanical systems, e.g., rigid and flexible robots, also to soft robots [8].

We further explore this direction by extending to continuum soft robots the iterative learning regulator of [9]. The controller, which consists of a PD action and a feedforward term, guarantees asymptotic regulation of the configuration variables provided that the proportional gain is large enough.

II. Dynamic Model

Consider a continuum soft robot modeled as

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) + \boldsymbol{K}\boldsymbol{q} + \boldsymbol{D}\dot{\boldsymbol{q}} = \boldsymbol{\tau}, \qquad (1)$$

where $\boldsymbol{q} \in \mathbb{R}^n$ denotes the configuration vector, $\boldsymbol{M}(\boldsymbol{q}) > 0$ is the $n \times n$ robot inertia matrix, $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$ collects Coriolis and centrifugal terms, and $\boldsymbol{G}(\boldsymbol{q}) = \nabla_{\boldsymbol{q}} U_g(\boldsymbol{q})$ is the gravity vector, with $U_g(\boldsymbol{q})$ the robot gravitational potential energy. Finally, $\boldsymbol{K}\boldsymbol{q}$ and $\boldsymbol{D}\dot{\boldsymbol{q}}$ model, respectively, elastic and dissipative effects, and $\boldsymbol{\tau}$ is the control input.

The above model verifies the following property inherited from rigid robots with revolute joints [10].

Property 1. *For all $\boldsymbol{q} \in \mathbb{R}^n$, there exist constants $\alpha_{U_g}, \alpha_{\boldsymbol{G}}, \alpha_{\boldsymbol{\nabla}^2(U_g)} > 0$ such that*

$$\|U_g(\boldsymbol{q})\| \leq \alpha_{U_g},\ \|\boldsymbol{G}(\boldsymbol{q})\| \leq \alpha_{\boldsymbol{G}},\ \left\|\nabla^2_{\boldsymbol{q}} U_g(\boldsymbol{q})\right\| \leq \alpha_{\boldsymbol{\nabla}^2(U_g)}.$$

The last inequality also implies that $\boldsymbol{G}(\boldsymbol{q})$ is globally Lipschitz.

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III. Control Law

In this section, we present our main result, i.e., a learning-based regulator that iteratively steers the robot configuration to any desired target.

Consider the control law

$$\boldsymbol{\tau} = \gamma \boldsymbol{K}_P \left(\boldsymbol{q}_d - \boldsymbol{q}\right) - \boldsymbol{K}_D \dot{\boldsymbol{q}} + \boldsymbol{\upsilon}_{i-1}, \qquad (2)$$

where $i \in \mathbb{N}_{>0}$ denotes the iteration index, $\boldsymbol{q}_d$ is a desired configuration, $\gamma > 2$ is the learning gain, $\boldsymbol{K}_P > 0$, $\boldsymbol{K}_D \geq 0$ are symmetric (diagonal) $n \times n$ gain matrices, and $\boldsymbol{\upsilon}_{i-1}$ is a feedforward action to be learned. The first iteration ($i = 1$) is executed with $\boldsymbol{\upsilon}_0 = \boldsymbol{0}$. As soon as the closed-loop system (1)–(2) reaches a new equilibrium configuration $\boldsymbol{q}_i$, the term $\boldsymbol{\upsilon}_{i-1} \in \mathbb{R}^n$ is updated as

$$\boldsymbol{\upsilon}_i = \gamma \boldsymbol{K}_P \left(\boldsymbol{q}_d - \boldsymbol{q}_i\right) + \boldsymbol{\upsilon}_{i-1}. \qquad (3)$$

Theorem 1. *Suppose that*

$$\boldsymbol{K}_P > (\lambda_{\max}(\boldsymbol{K}) + \alpha_{\boldsymbol{\nabla}^2(U_g)})\boldsymbol{I}_n, \qquad (4)$$

then, for all $i \in \mathbb{N}_{>0}$, the closed-loop system (1)–(2) *has a globally asymptotically stable equilibrium at $\boldsymbol{q}_i$. In addition,* (3) *guarantees*

$$\lim_{i \to \infty} \boldsymbol{q}_d - \boldsymbol{q}_i = \boldsymbol{0}.$$

Proof. Arguments similar to those of [11] show that, for all $i \in \mathbb{N}_{>0}$ and initial conditions, under (4) the trajectories of (1)–(2) are bounded and converge asymptotically to the unique solution of

$$\boldsymbol{G}(\boldsymbol{q}) + \boldsymbol{K}\boldsymbol{q} = \gamma \boldsymbol{K}_P \left(\boldsymbol{q}_d - \boldsymbol{q}\right) + \boldsymbol{\upsilon}_{i-1}. \qquad (5)$$

Let $\boldsymbol{e}_i := \boldsymbol{q}_d - \boldsymbol{q}_i$ denote the steady state tracking error of the i-th iteration, being $\boldsymbol{q}_i$ the solution of (5). Combining (3) with (5) results in $\boldsymbol{\upsilon}_i = \boldsymbol{G}(\boldsymbol{q}_i) + \boldsymbol{K}\boldsymbol{q}_i$, which implies by prop. 1 and after some computations,

$$\|\boldsymbol{\upsilon}_i - \boldsymbol{\upsilon}_{i-1}\| \leq \left(\alpha_{\boldsymbol{\nabla}^2(U_g)} + \lambda_{\max}(\boldsymbol{K})\right)\left(\|\boldsymbol{e}_i\| + \|\boldsymbol{e}_{i-1}\|\right). \qquad (6)$$

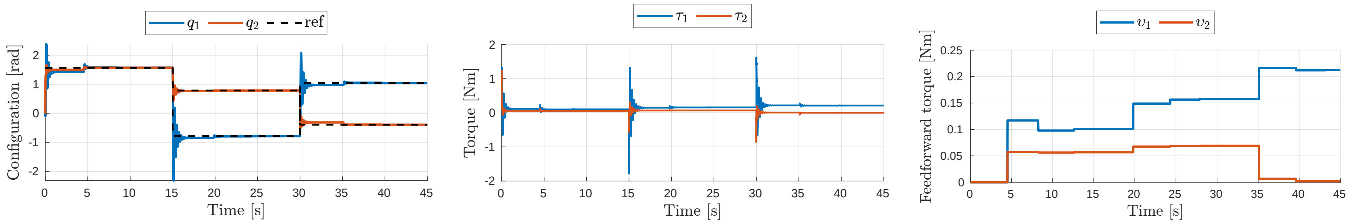
Now, (3) and (4) yield

$$\|\boldsymbol{\upsilon}_i - \boldsymbol{\upsilon}_{i-1}\| > \gamma \left(\lambda_{\max}(\boldsymbol{K}) + \alpha_{\boldsymbol{\nabla}^2(U_g)}\right)\|\boldsymbol{e}_i\|. \qquad (7)$$

Thus, from (6)–(7) we have $\|\boldsymbol{e}_i\| + \|\boldsymbol{e}_{i-1}\| > \gamma\|\boldsymbol{e}_i\|$, which for $\gamma > 2$ guarantees contraction of the tracking error as the iteration index i tends to ∞. □

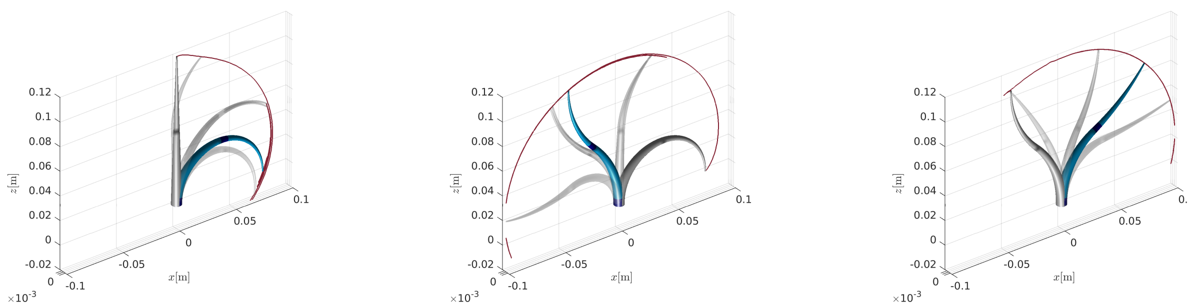
Remark 1. *At the expense of degrading performance, the control law does not require velocity measures, i.e., $\boldsymbol{K}_{\mathrm{D}}$ can be set to zero. In addition, only upper bounds associated with the potential field are needed.*

Remark 2. *The above result can be easily extended to soft robots with nonlinear elastic and dissipative forces, i.e., $\boldsymbol{K}(\boldsymbol{q})$ and $\boldsymbol{D}(\boldsymbol{q},\dot{\boldsymbol{q}})$.*



(a) (b) (c)

Figure 1. Simulation results. Time evolution of the configuration variables (a), control inputs (b) and learned feedforward terms (c).



(a) $t \in [0; 15)$ [s] (b) $t \in [15; 30)$ [s] (c) $t \in [30; 45]$ [s]

Figure 2. Simulation results. Stroboscopic view of the robot motion in its workspace in the three time windows defined by the reference trajectory in eq. (8). Initial and final configurations are depicted in dark gray and blue, respectively. A red line illustrates the trajectory traced by the robot tip.

IV. Simulation Results

Consider a soft robot with two actuated segments moving in a vertical plane, i.e., under gravity, oriented so that its tip points upward in the straight configuration. Under the piece-wise constant curvature (PCC) assumption [8], each segment has only one degree of freedom, i.e., its curvature q_j, $j = 1, 2$, so that $\boldsymbol{q} = (q_1\ q_2)^T \in \mathbb{R}^2$. Each segment has mass $m_j = 0.335\,[\mathrm{kg}]$ and length $l_j = 0.06\,[\mathrm{m}]$. Stiffness and damping are uniformly distributed and equal to $\boldsymbol{K} = 0.05 \cdot \boldsymbol{I}_2$ [Nm/rad] and $\boldsymbol{D} = 0.008 \cdot \boldsymbol{I}_2$ [Nms/rad], respectively. The control gains are taken as $\gamma = 4$, $\boldsymbol{K}_P = 0.2 \cdot \boldsymbol{I}_2$[Nm/rad] and $\boldsymbol{K}_D = 0.001 \cdot \boldsymbol{I}_2$ [Nms/rad]. The robot starts at rest in the straight configuration, and the simulation runs for $45\,[\mathrm{s}]$. The piece-wise constant commanded reference is (in [rad])

$$\boldsymbol{q}_d(t) = \begin{cases} (\pi/2;\ \pi/2)^T; & t \in [0; 15) \quad [\mathrm{s}] \\ (-\pi/4;\ \pi/4)^T; & t \in [15; 30) \ [\mathrm{s}] \\ (\pi/3;\ -\pi/8)^T; & t \in [30; 45] \ [\mathrm{s}] \end{cases} . \tag{8}$$

Figure 1(a) shows the closed-loop evolution of the configuration variables for reference (8). For all the three commanded targets, the controller, whose output is shown in Fig. 1(b), quickly regulates the curvatures to the desired value. The small derivative action leads to poor transient performance. Nonetheless, the closed system remains always stable, as expected. In addition, the control law learns the potential forces at the desired configuration in very few iterations, as illustrated by the evolution of the feedforward term in Fig. 1(c). Finally, Figure 2 shows a stroboscopic plot of the robot motion in its workspace, see also the accompanying video.

V. Conclusions and Future Works

In this work, we proposed an iterative learning regulator for continuum soft robots. The control law requires minimal system knowledge for implementation and achieves global asymptotic stabilization of the closed-loop system if the proportional gain is sufficiently large. Future works will be devoted to the extension of the control law to underactuated soft robots and its experimental validation.

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